Physics | Oct 22 |

Magnetic Flux density.

$$\oint_c \vec{H} \cdot d\vec{l} = I_{enclosed} = \oint_s \vec{J} \cdot d\vec{S}$$

$$\oint_s (\vec{\nabla} \times \vec{H}) \cdot d\vec{S} = \oint_s \vec{J} \cdot d\vec{S}$$

$$\therefore \quad \vec{\nabla} \times \vec{H} = \vec{J}$$

$$\oint_s \vec{D} \cdot d\vec{S} = q_{enclosed} \Rightarrow \vec{\nabla} \cdot \vec{D} = \rho_s$$

$$\oint_c \vec{E} \cdot d\vec{l} = 0 \Rightarrow \vec{\nabla} \times \vec{E} = 0$$

$$\oint \vec{H} \cdot d\vec{l} = I_{enclosed} \Rightarrow \vec{\nabla} \times \vec{H} = \vec{J}$$

Recall:

$$\vec{D} = \varepsilon_o \vec{E}$$
$$\vec{B} = \mu_o \vec{H}$$

magnetic permibility in vacuum

$$\mu_o = 4\pi(10^{-7})$$

Note:

$$\frac{1}{\sqrt{\varepsilon_o \mu_o}} = c$$

speed of light in vacuum.

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

net magnetic sources
(there can not be a north or south pole by themselves)

$$\oint_V (\vec{\nabla} \cdot \vec{B})\, dV = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Summary

$$\left.\begin{array}{ll} \vec{\nabla} \cdot \vec{D} = \rho_s & \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = 0 & \vec{\nabla} \times \vec{H} = \vec{J} \end{array}\right\}$$

Valid for static cases.

We know

$$\vec{\nabla} \times E = 0 \quad \& \quad \vec{E} = \vec{\nabla}(-V)$$

$$(\vec{\nabla} \times \vec{\nabla})(\text{anything}) = 0$$

V is the potential, it is a scaler because the charge is a scaler.

$$\vec{\nabla}(V + k) = \vec{\nabla}(V)$$

$$\vec{\nabla}\cdot\vec{B}=0 \quad \& \quad \vec{B}=\vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})=0$$

$\vec{A}$ is called a potential vector, b/c the current is a vector.

$$\vec{\nabla}\times(\vec{A}+\vec{\nabla}(\text{something}))=\vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\cdot\vec{A}=0 \qquad (\text{Lorentz gauge})$$

This is an assumption that makes calculations easier.

We can relate V to $\vec{E}$ & $\vec{D}$ & to the sources ρ_s.

$$\vec{E}=-\vec{\nabla}V \quad \vec{\nabla}\cdot\vec{D}=\rho_s \quad \varepsilon_0\vec{E}=\vec{D}$$

$$\vec{\nabla}\cdot\vec{E}=-\vec{\nabla}^2 V$$

$$=\frac{\rho_s}{\varepsilon_0}$$

$$\therefore \quad -\vec{\nabla}^2 V=\frac{\rho_s}{\varepsilon_0}$$

$$\vec{B}=\vec{\nabla}\times\vec{A} \qquad \vec{\nabla}\times\vec{H}=\vec{J}$$

$$\cancel{\vec{B}} \qquad \vec{\nabla}\times\vec{B}=\vec{\nabla}\times(\vec{\nabla}\times\vec{A})=\mu_0\vec{J}$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}^2\vec{A} = \mu_0\vec{J} \qquad \text{(vector identity)}$$

$$\vec{\nabla}^2\vec{A} = -\mu_0\vec{J}$$

One can show...

$$\vec{A}(r_p) = \int_L \frac{\mu_0 I}{4\pi}\frac{d\vec{l}_s}{|r_p - r_s|}$$

$$\vec{\nabla}\times\vec{A} = \vec{\nabla}_p \times \vec{A}(r_p)$$

$$\vec{\nabla}_p = \left\langle \frac{\partial}{\partial x_p}, \frac{\partial}{\partial y_p}, \frac{\partial}{\partial z_p} \right\rangle$$

$$\vec{\nabla}\times\vec{A} = \int_L \frac{\mu_0 I}{4\pi}\vec{\nabla}_p \times \left(\frac{d\vec{l}_s}{|\vec{r}_p - \vec{r}_s|}\right)$$

$$\left(\vec{\nabla}_p \frac{1}{|\vec{r}_p - \vec{r}_s|}\right) \times dl_s + \frac{1}{|\vec{r}_p - \vec{r}_s|}\underbrace{\vec{\nabla}_p \times d\vec{l}_s}_{0}$$

$$\vec{\nabla}\times\vec{A} = -\int_L \frac{\mu_0 I}{4\pi}\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3} \times d\vec{l}_s$$

$$= +\int_L \frac{\mu_0 I}{4\pi} d\vec{l}_s \times \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

$$\vec{A} = \int \frac{\mu_0}{4\pi} \vec{K} \frac{ds}{|\vec{r_p} - \vec{r_s}|}$$ for surface currents

$$\vec{A} = \int \frac{\mu_0}{4\pi} \vec{J} \frac{d\vec{V}}{|\vec{r_p} - \vec{r_s}|}$$ for volume currents.

Magnetic forces.

Electrostactic forces are generated by "static" charges and act on "static" charges

Magnetostatic forces are generated by "static" currents and act on "static" currents.

moving charges.

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{F_m} = (q\vec{u}) \times \vec{B}$$

$$\vec{F_{net}} = q(\vec{E} + \vec{\mu} \times \vec{B})$$ (Lorentz force)

note: $\vec{\mu}$: velocity of charged particle.

Force on a small current-carrying wire

$$I = \frac{\Delta q}{\Delta t} = \frac{\rho S \Delta l}{\Delta t}$$ $\frac{\Delta l}{\Delta t}$: velocity of charges

$$I = \rho S |\vec{u}|$$

$$J = \frac{I}{S} = \rho |\vec{u}|$$

$$\vec{J} = \rho \vec{u}$$

$$I\,d\vec{l} = \vec{J} \cdot d\vec{V}$$

$$= \rho S \cancel{\vec{u}}\, u\, d\vec{l} = \rho S \vec{u} \cdot dV$$

$$= (\rho\, dV)\, \vec{u}$$

$$= dq\, \vec{u}$$

Small peice of wire $d\vec{l}$ with current I

$$d\vec{F}_m = (dq\, \vec{u}) \times \vec{B} = I\, d\vec{l} \times \vec{B}$$

$$\overrightarrow{F_{mag}} = \int_L I\, d\vec{l} \times \vec{B}$$

Nota Bena This is the force on wire. This wire is <u>not</u> the source $\vec{B}$.

$$\vec{H} = \int \frac{I}{4\pi}\, d\vec{l}_s \times \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

$$= \int (\vec{K}\, dS) \times \vec{B}$$

$$= \int (\vec{J}\, dV) \times \vec{B}$$

Fources between current carrying wire

$d\vec{l}_s$: little peice of source wire

$d\vec{l}_p$ = little peice of target wire

(now calculate the force on the target wire)

$$d(d\vec{F}_p) = I_p\, d\vec{l}_p \times d\vec{B}_s$$

$$d\vec{B}_s = \mu_0\, d\vec{H}_s = \mu_0 \frac{I_s}{4\pi}\, d\vec{l}_s \times \left[\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}\right]$$

$$d\vec{F}_p = \int_{l_s} I_p\, d\vec{l}_p \times \left(\frac{\mu_0 I_s}{4\pi}\, d\vec{l}_s \times \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}\right)$$

$$\vec{F}_p = \int_{l_p}\int_{l_s} I_p\, d\vec{l}_p \times \left(\frac{\mu_0 I_s}{4\pi}\, d\vec{l}_s \times \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}\right)$$

$$= \frac{\mu_0 I_s I_p}{4\pi} \int_{l_p} d\vec{l}_p \times \left[\int d\vec{l}_s \times \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}\right]$$